On existential quantification and hyperspaces for logic on words

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Two approaches have been remarkably effective in the study of formal languages: the algebraic one, and the logical one. Whereas the former relies on the notions of recognition by a finite monoid and of syntactic monoid of a language, the latter is based on a semantic on finite words.

In particular, a fundamental rôle is played by *regular* languages, which can be characterised as those whose syntactic monoid is finite, or as those languages definable by means of a monadic second-order formula.

In the last years, a deep connection between Stone duality and the algebraic theory of formal languages has been exploited. Following these lines, in [1] we have investigated *beyond the regular context* the effect — at the level of the recognising objects — of applying a layer of (first-order) existential quantifier, i.e. passing from a language defined by a formula $\Phi(x)$ to the language defined by the formula $\exists x.\Phi(x)$.

In order to deal with non-regular languages, it is necessary to reconsider the notion of recognising object. This leads us to syntactic spaces for a language which are in particular Stone spaces. In this framework, it turns out that the existential quantification essentially corresponds to the classical topological construction of the *hyperspace* (or Vietoris space) of a topological space.

The operation on spaces corresponding to the existential quantification on languages can be regarded as a functor on the category of recognising objects. If time allows, we will show how the natural binary extension of this functor generalises a classical construction related to the concatenation of languages, namely the *Schützenberger product* of two monoids.

References

[1] M. Gehrke, D. Petrişan, and L. Reggio. The Schützenberger product for syntactic spaces. arXiv:1603.08264 [cs.LO], 2016.